Engineering Measurements

Chapter Two

Analysis of experimental data

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E n g i n e e r i n g m e a s u r e m e n t s

Method of Least Squares

Suppose we have a set of observations $x_1, x_2, \ldots, x_n$. *The sum of the squares of their* deviations from some mean value is:

$$S = \sum_{i=1}^{n} (x_i - x_m)^2$$

Now, suppose we wish to minimize *S with respect to the mean value* x_m.

We set

$$\frac{\partial S}{\partial x_m} = 0 = \sum_{i=1}^{n} -2(x_i - x_m) = -2\sum_{i=1}^{n} x_i - n x_m$$

where *n is the number of observations. We find that*

$$x_m = \frac{1}{n}\sum_{i=1}^{n} x_i$$

This is the simplest example of method of least square

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Method of Least Squares

Curve fitting

❖In experimental work, we change some variables and measure others. The first type of variables are called independent variables (x) and the other kind can be presented in is called dependent variables (y). So, the collected data as arranged pairs (x,y)

❖ It is more convenient from the mathematical point of view to represent the relation between the arranged pairs as: y=f(x)

❖The process where a function is used to describe a set of points pairs: (x_1,y_1), (x_2,y_2),…. (x_n,y_n) is called curve fitting

❖When we have the case of precise points, the fitting is called **interpolation**.

❖When we have the case of imprecise points, the fitting is called **regression**.

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Method of Least Squares

Curve fitting

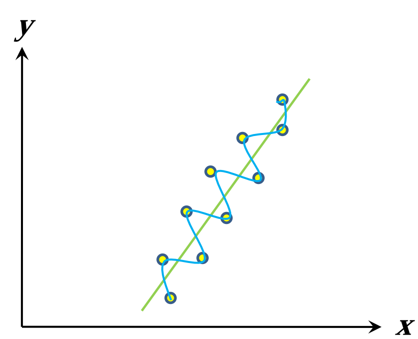
❖to estimate the value between the points, we use a polynomial:

$$F(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$$

for (n+1) data points $(x_0,f(x_0))$, $x_1,f(x_1)$, …., $x_n,f(x_n)$.

❖The simplest form of interpolation is the linear :

$$F(x) = y = ax + b$$



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Method of Least Squares

Curve fitting

❖To minimize the quantity:

$$S=\sum_{i=1}^{n}(y_i-ax-b)^2$$

❖Now set the derivatives with respect to (a) and (b) to zero. With some mathematical manipulation, we will have :

$$a=\frac{n\sum x_i y_i-\left(\sum x_i\right)\left(\sum y_i\right)}{n\sum x_i^2-\left(\sum x_i\right)^2} \qquad b=\frac{\left(\sum y_i\right)\left(\sum x_i^2\right)-\left(\sum x_i y_i\right)\left(\sum x_i\right)}{n\left(\sum x_i^2\right)-\left(\sum x_i\right)^2}$$

❖We designate the value of (y=ax+b) as: $\hat{y}=ax+b$

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Method of Least Squares

Curve fitting

❖The standard error can be as

$$\text{Standard error}=\sqrt{\frac{\sum(y_i-\hat{y}_i)^2}{n-2}}=\sqrt{\frac{\sum(y_i-ax_i-b)^2}{n-2}}$$

❖The method of least squares insure the minimum value of error in curve fitting:

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Analysis of experimental data

Example [5]:

From the following data obtain *y as a linear function* of *x using the method of least squares*

x_i	y_i
1.0	1.2
1.6	2.0
3.4	2.4
4.0	3.5
5.2	3.5

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Analysis of experimental data

Example [5]:

Solution

We need to find y =ax+b. to do that, we need to find a and b:

$$a = \frac{(5)(44.76)-(15.2)(12.6)}{(5)(58.16)-(15.2)^2} = 0.540 \qquad b = \frac{(12.6)(58.16)-(44.76)(15.2)}{(5)(58.16)-(15.2)^2} = 0.879$$

So: y = 0.540x+0.879

x_i	y_i	$x_i y_i$	x_i^2
1.0	1.2	1.2	1.0
1.6	2.0	3.2	2.56
3.4	2.4	8.16	11.56
4.0	3.5	14.0	16.0
5.2	3.5	18.2	27.04
∑=15.2	∑=12.6	44.76	58.16